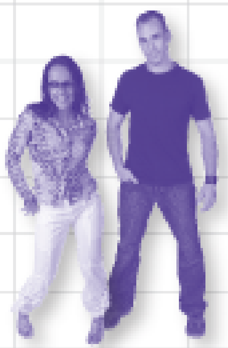
A Brain-Friendly Guide

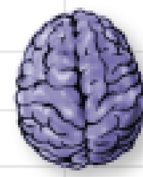
Includes Pre-Algebra Review

Head First Algebra

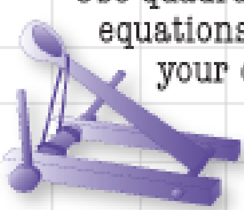
Who's got mad algebra skills? YOU be the judge



Load algebra straight into your brain

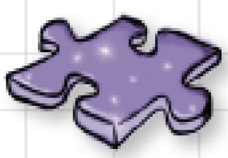


Use quadratic equations to power your catapult



Avoid getting ripped off by solving linear equations

Bend your mind around dozens of puzzles & exercises



Take a ride with algebra in the real world

Tracey Pilone, M.Ed.
& Dan Pilone

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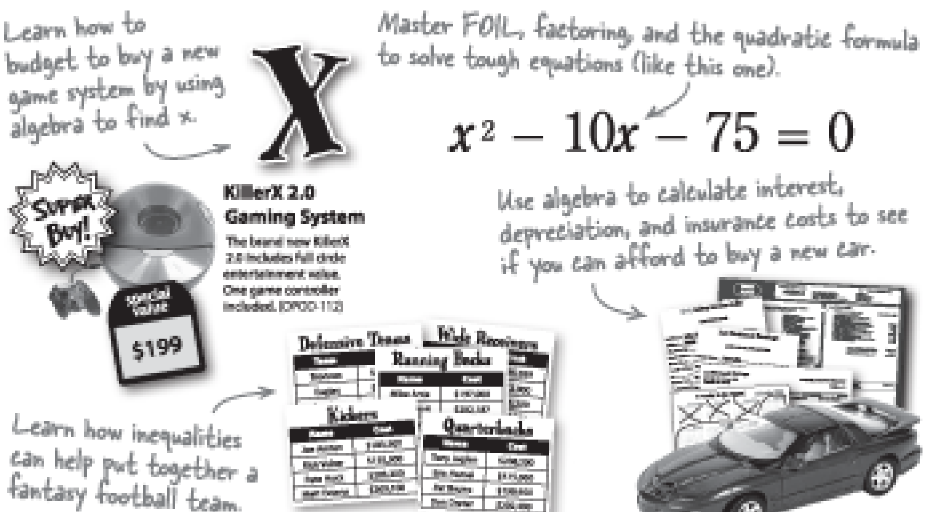
Head First Algebra

Algebra/Math

What will you learn from this book?

Tired of struggling just to get a C- in your college algebra class? Do you need to pass high school algebra to get your cell phone back? If you need to get algebra in your brain, then *Head First Algebra* is designed for you. Full of engaging stories and practical, real-world explanations, you'll learn everything from natural numbers to exponents to solving systems of equations and graphing polynomials.



Why does this book look so different?

We think your time is too valuable to spend struggling with new concepts. Using the latest research in cognitive science and learning theory to craft a multi-sensory learning experience, *Head First Algebra* uses a visually rich format designed for the way your brain works, not a text-heavy approach that puts you to sleep.

"The book is driven by excellent examples from the world in which students live. No trains leaving from the same station at the same time moving in opposite directions."

—*Herbert Tracey, Instructor of Mathematical Sciences, Loyola University*

"*Head First Algebra* was an engaging read. The book did a fantastic job of explaining concepts and taking the reader step-by-step through solving problems."

—*Shannon Stewart, Math Teacher*

"The way this book presents information is so conversational and intriguing it helps in the learning process. It truly feels like you're having a conversation with the author."

—*Amanda Borcky*

US $29.99 CAN $29.99
ISBN: 978-0-596-51486-0
52999
9 780596 514860

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Head First Algebra

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Printed in the United States of America.

Published by O'Reilly Media, Inc., 1005 Gravenstein Highway North, Sebastopol, CA 95472.

O'Reilly Media books may be purchased for educational, business, or sales promotional use. Online editions are also available for most titles (*safari.oreilly.com*). For more information, contact our corporate/institutional sales department: (800) 998-9938 or *corporate@oreilly.com*.

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Cover Designers: Louise Barr, Steve Fehler
Production Editor: Brittany Smith
Indexer: Julie Hawks
Page Viewers: Vinny and Nick

Printing History:

December 2008: First Edition.

No variables were harmed in the making of this book.

This book uses RepKover™, a durable and flexible lay-flat binding.

ISBN: 978-0-596-51486-0

[M]

4 exponent operations

Podcasts that spread like the plague

(that's a good thing...)

Could you multiply that again? Could you multiply that again?

There's another way to express multiplication that's repeated over and over and over again, without just repeating yourself. **Exponents** are a way of **repeating multiplication**. But there's more to exponents, including some smaller-than-usual numbers (and we don't just mean fractions). In this chapter, you'll brush up on **bases**, **roots**, and **radicals**, all without getting arrested for any sit-in protests. And, as usual, ***zero*** and ***one*** come with their own problems... so jump into a podcasting exponentiation extravaganza.

Addie's got a podcast

I've been producing my own Podcast, but now I need better quality gear... and new equipment is expensive!

Addie, podcast producer extraordinaire.

Ahem, eccentric

Addie podcasts about ~~crazy~~ celebrities.

Addie's been getting a lot more listeners lately. To take it to the next level, she needs new equipment to deliver an even better podcast... but that takes a lot of cash.

Addie's got a website to host her podcasts, and she wants to advertise on her site to raise funds for new gear. She's lined up some possible sponsors, but they won't help out until Addie proves she can get some real traffic on her site. Addie needs to:

- ...monitor the daily hits on her website over the next 2 weeks.

- ...prove her site can generate at least **5,000,000 hits** in the next 2 weeks!

Wow, that's a lot.

Addie's computer. Can you say 1987?

Let's mobilize Addie's listeners

Addie knows she's got big fans. Here's a letter she's worked up to send out to her **3 top listeners**:

Dear TOP 3 Listeners,

I'm trying to get some sponsors for the podcast so we can buy some new equipment. There will be some advertisers tracking the hits to the site over the next two weeks, and we need to have 5,000,000 hits in a single day.

Please make sure that you visit the site today and send this email out to **3 more people.** If everyone keeps visiting the site and emailing 3 new people for 14 days, I think we'll make it!

Thanks,

Addie from the StarTalk Podcast

Sharpen your pencil

Write down the equation to figure out how many hits Addie will get to by the end of 14 days, assuming each of her 3 top listeners tells 3 more people each day to visit her site. Don't worry about solving the equation for now.

....

....

Sharpen your pencil Solution

Your job was to write down the equation to figure out how many hits Addie will get to by the end of 14 days, assuming each of her 3 top listeners tells 3 more people each day to visit her site.

No. of hits = 3 times 3, every day for 14 days

Each day, we multiply by 3, since 3 more people get involved.

hits = 3•3•3•3•3•3•3•3•3•3•3•3•3•3

Wow, that's a lot of threes... and hard to keep up with.

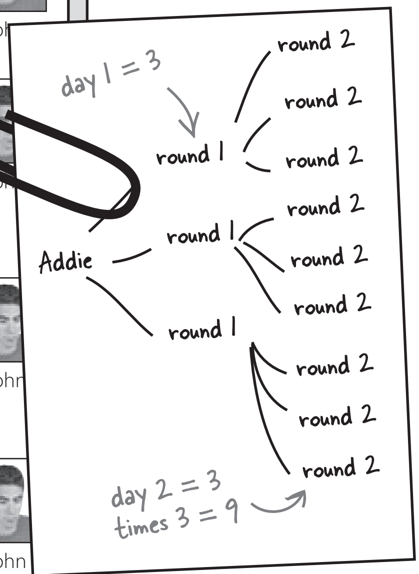
IM Chat: Exponents

Jane: That equation is HUGE.

John: Well, it's 14 days, and if everybody who gets the email hits the site, that's 3 times 3 for each day.

Jane: Hmm, I don't really get why...

John: Here's an image. Here are the first 2 rounds.



Jane: Ok - so the number of hits on day two is 2 threes multiplied together. And 14 days is 14 threes multiplied together.

John: Exactly.

Jane: I keep trying to work this out with my calculator, but I keep losing count of how many threes I've put in.

John: There's a better way to do that. Do you know anything about exponents?

Jane: What?

John: Exponents are a shorthand way to show multiplication over and over again. It works with your calculator, too. Two numbers, not a row of 14.

Jane: ...

I'm lost. What are you talking about?

Exponents Up Close

Exponents are a special notation used to express repeated multiplication. That's just what we need to figure out Addie's number of hits without counting a lot of threes: a way to show multiplying by 3 over and over.

When you see a number with an exponent, it looks like this:

exponent

$$x^{a} = x \cdot x \cdot x \ldots \cdot x$$

base

This means multiply x by itself "a" times.

The base is the number being multiplied (in Addie's case, 3), and the exponent is the number of times you repeat it (in Addie's case, 14). Those two numbers are all you need to put in your calculator, and you'll get the answer.

EQUATION CONSTRUCTION

Rewrite Addie's equation using exponent notation, and solve (using a calculator to get the number is a good idea).

This is the equation for Addie's website hits.

hits = 3•3•3•3•3•3•3•3•3•3•3•3•3•3

Equation Construction Solution

Your job was to rewrite Addie's equation using exponent notation, and solve.

That's 14 threes!

hits = $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

3 is the base because it's the number being multiplied.

hits = 3^{14}

14 is the exponent because that's how many times you multiply 3 by itself.

$h = 3^{14} = 4{,}782{,}969$

Over 4 million hits on the 14th day. That's awesome... but not enough.

That's not going to cut it. I need 5,000,000 hits to get sponsorship. I need some help!

Whaddup, girl? I can help you out... I've got tons of friends, you know. Have you seen my Facebook page?

Addie's brother, Alex

Can Addie and Alex get enough hits?

Alex has offered to send off another round of emails for Addie. He'll start with 3 friends, just like she did, and try to help get 5,000,000 hits in 14 days.

To figure out the total number of hits, you'll need to figure out how to add up both groups that Addie's working with. In chapter 2, you combined like terms to help Paul on his road trip, and this is the same idea. You may remember from chapter 2 that a **term** is any part of an equation held together with multiplication or division. Since an exponent is just a shorthand version of multiplication, that means ***exponential terms with the same base and the same exponent are like terms.***

With exponents, you can combine terms that have the same base. Let's try that out and see how it works:

Math Magnets

Write the new equation for the number of hits that Addie and Alex will get. Will she reach 5,000,000 now?

Addie contacts 3 friends...

Now Alex contacts 3 friends, too.

Use h to be the number of hits.

$$\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_$$

Exponents with the same base are like terms.

$$h = 2(\_\_\_\_)$$

That's the total number of hits.

$$h = \_\_\_\_$$

Did they get 5,000,000 hits? ....

14 | No | 14 | 3 | 14 | 9,565,938 | + | 3 | 3 | h | 5,000,000 | Yes

Math Magnets Solution

Write the new equation for the number of hits that Addie and Alex will get. Will she reach 5,000,000 now?

Addie contacts 3 friends...

Now Alex contacts 3 friends, too.

$$h = 3^{14} + 3^{14}$$

Use h to be the number of hits.

$$h = 2(3^{14})$$

Exponents with the same base are like terms.

$$h = 9{,}565{,}938$$

That's the total number of hits.

Did they get 5,000,000 hits? Yes

5,000,000

No

Wait. Why is that $2(3^{14})$ and not $(3^{14})^2$?

Because $(3^{14})^2$ is multiplication, not addition.

A term is something held together by multiplication, which means that the entire exponential term is treated as a group.

3^{14} is one term, not two.

When you group two like terms together, you're adding those terms together. But if you take those two terms and use exponents (that ^{2} at the end of $(3^{14})^2$), then you're multiplying, and that's not what we want. Look:

$$3^{14} + 3^{14} = 2(3^{14})$$

This term plus this term means two of the same term.

Remember – the exponent means you multiply.

$$(3^{14})^2 = 3^{14} \cdot 3^{14}$$

This is just another way to write multiply 3^{14} by itself 2 times.

Ok, so $(3^{14})^2$ isn't the right answer, but how would you work that sort of problem out, anyway? I guess we could write out a bunch of 3's in a big line?

Well, that would work, but that's a lot of threes...

Writing out the multiplication by hand will work; it's just not very convenient. Look how long this thing turns out to be:

So many 3's...

$$(3^{14})^2 = 3^{14} \cdot 3^{14}$$
$$=(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)$$
$$=3^{\boxed{}}$$

What's the final exponent? Count them up and fill this box in.

But look, there's a pattern! Here's what this means:

$$(x^a)^b = x^{a \cdot b}$$

Multiply 2•14, it's the same thing you got when we did it the long way.

We combined those exponents with like bases before. Is there anything else we can do with like bases?

Exponents with the same base are LIKE TERMS.

That means they can be added, subtracted, multiplied, and divided.

Try division:

$$\frac{3^{14}}{3^{12}} = \frac{3 \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot 3}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}$$
$$=3 \cdot 3$$

If you write it out, you can see how many factors you can divide out.

Sharpen your pencil

Write the general form to combine like terms. We've gone ahead and done the first one for you.

$3^{14} + 3^{14} = x^a + x^a = 2x^a$

$\frac{3^{14}}{3^{12}} =$ ______________ $=$

$(3^{14})^2 =$ $\qquad\qquad$ $=$

$3^{14} \cdot 3^2 = x^a \cdot x^b =$

Go back to writing the whole thing out if you need to, but you can figure it out!

Sharpen your pencil Solution

Your job was to write the general form to combine like terms.

For division, you can just subtract the denominator exponent from the numerator exponent (it's just a quick way to figure out which ones you divided out).

$$3^{14} + 3^{14} = x^a + x^a = 2x^a$$

$$\frac{3^{14}}{3^{12}} = \frac{x^a}{x^b} = x^{a-b}$$

$$(3^{14})^2 = (x^a)^b = x^{ab}$$

$$3^{14} \cdot 3^2 = x^a \cdot x^b = x^{a+b}$$

You just multiply the two exponents together...

3•3•3•3•3•3•3•3•3•3•3•3•3•3•3•3

Start by doing it the long way – it's 16 threes.

Alex is flaking out on his sister

Addie,

Sorry, but I totally forgot to send out that email for your podcast until today. I hope you still make it.

Alex

Alex didn't send out any emails to his friends until the third day. That means he only has 12 days to get the word out. Will Addie still make it?

Sharpen your pencil

Now will Addie make it? Figure out how many hits she'll get since Alex's email only has 12 days to work, not 14.

Write the new equation and solve it:

What is the general form of this equation?

Use x and y as bases and a and b as exponents.

Do the exponential terms have the same base? **Yes** **No**

Circle one

Will Addie still have enough hits to make 5,000,000? **Yes** **No**

Sharpen your pencil Solution

Now will Addie make it? Figure out how many hits she'll get since Alex's email only has 12 days to work, not 14.

Alex has two fewer days, so that's 12.

Write the new equation and solve it: $h = 3^{14} + 3^{12}$

Addie's email is the same.

$h = 4{,}782{,}969 + 531{,}441$

$h = 5{,}314{,}410$

What is the general form of this equation? $h = x^a + x^b$

Use x and y as bases and a and b as exponents.

These can't really be combined easily. Since they don't have the same exponent, they're NOT like terms.

Do the exponential terms have the same base? **Yes** **No**

But just by 314,410 hits. It's pretty close.

Will Addie still have enough hits to make 5,000,000? **Yes** **No**

there are no Dumb Questions

Q: : Why use exponents and not just multiplication?

A: Because it can save you a bunch of work. Writing out a value to multiply over and over again is tedious and leads to error. And when the numbers start to get really big (like an exponent of 14), they are just impossible to deal with otherwise.

Q: Why do you need the same base and the same exponent to do addition and subtraction?

A: Because they need to be **like terms**. Remember that exponents are a shorthand for multiplication. Because of the order of operations, you can't add two multiplication expressions together without doing the multiplications first... unless you've got like terms. If the expressions are like terms, then you can collect them together into a single term. That's exactly what adding exponential terms with the same base and exponent does**!**

Q: Where are exponents in the order of operations?

A: They're second. Since exponents are just a more powerful form of multiplication, they go ***before*** multiplication. So it's parentheses, exponents, ***then*** multiplication and division.

Q: How do you work with exponents with different bases?

A: We're going to be looking at those next. But fair warning, there's not much you can do to make those problems simpler. If you have two bases, then there are two things that need to be multiplied, divided, or whatever. There's not a good way to combine terms like that since you've got to keep track of both bases separately.

Q: What happens if I'm dividing exponential terms and the exponent becomes negative?

A: Great question! When you divide exponential terms, you subtract the exponents. This means that you could end up with a negative exponent. The good news is that this is easy to deal with. A negative exponent just means 1 over the positive exponential term. So:

2^{-1} is 1/2,
x^{-25} is $1/x^{25}$

...and so on.

There's always a villain...

The Movie Podcast heard about Addie's plan to increase subsribers, and they don't like it. The sponsorship Addie's trying to get... well, it's money out of the Movie Podcast's pocket. So now they're fighting back.

Dear **Top 4** Movie Podcast subscribers,

The StarTalk Podcast is trying to steal advertisers from us! If they hit 5,000,000 hits in the next 10 days, our sponsor is going to leave our show.

We need to fight back! Don't go to the Startalk page, and email 4 Startalk users, telling them not to either. If everybody emails 4 people, we'll pull enough hits so that she won't make it!

Addie's had a head start before this mail went out.

Thanks,

Movie Podcast

Every person that hits Movie Podcast's page instead of Startalk Podcast's page is taking away potential hits. What does this mean for Addie's chances to score a new sponsorship deal?

EQUATION CONSTRUCTION

Since Movie Podcast is going to take away hits, how many will be left? Is Addie going to make it or is she in trouble?

Write the new equation and solve it:

Don't forget about what Addie and Alex already did.

Will Addie still have enough hits to make 5,000,000? **Yes** **No**

Circle one

If No, how many more hits does Addie need to get to 5,000,000?

Write the equation in general form:

How many different bases are involved? **1** **2** **3**

How many different exponents? **1** **2** **3**

EQUATION CONSTRUCTION SOLUTION

Since Movie Podcast is going to take away hits, how many will be left? Is Addie going to make it, or is she in trouble?

Addie's orignial hits

Alex's email (two days late)

Write the new equation and solve it: $h = 3^{14} + 3^{12} - 4^{10}$

Movie Podcast's email: 10 days left and 4 emails each.

$h = 4{,}782{,}969 + 531{,}441 - 1{,}048{,}576$

Uh oh. Movie Podcast made enough of a dent to push Addie below the numbers she needs.

$h = 4{,}265{,}834$

Will Addie still have enough hits to make 5,000,000? **Yes** **No**

The number of hits she needs

Less what she has now (thanks to the folks at Movie Podcast)

If No, how many more hits does Addie need to get to 5,000,000? $5{,}000{,}000 - 4{,}265{,}834 = 734{,}166$

Addie needs to come up with over 700,000 new hits!

Write the equation in general form: $h = 3^{14} + 3^{12} - 4^{10}$

The new term has a different base AND a different exponent.

$h = x^a + x^b - y^c$

These are the same from earlier – same base but different exponent

How many different bases are involved? **1** **2** **3**

Since Addie and Alex sent theirs out to the same number of folks, they have the same base. Movie Podcast sent it out to more people with less time.

How many different exponents? **1** **2** **3**

That's why we have three different variables listed for the exponents. It also means that they can't be easily combined.

Since those terms have different bases, they can't be combined as variables, right?

Different bases = NOT like terms.

Terms with different bases are not like terms (regardless of the exponent). They just don't have anything in common. As exponential terms, they're not multiplying the same number, regardless of how many times.

As we saw earlier, they're only like terms if the base AND the exponent are the same.

You can't add exponents with different bases

If we just talk about the bit of Addie's equation that has two terms, it looks like this:

$$x^b - y^c = ?$$

This is just a piece of Addie's equation.

You know that you can't add or subtract these two because they're not like terms. Multiplication and division don't work either. Exponential terms being multiplied together just get written together, like this:

$$x^b\left(y^c\right) = x^b \cdot y^c = x^b y^c$$

These are all same thing, just written differently, but you can't combine them at all.

Why can't we just mush everything all together, like $(xy)^{bc}$?

The order of operations says exponents FIRST

You can't split up bases and combine different exponents because each base has to stay with its own exponent. The order of operations says that exponents go *before* multiplication. That means the exponents have to be **simplified** before they can be combined with something else.

OK

$$x^b(y^c) = x^b \cdot y^c = x^b y^c$$

These are all ok because the exponent stays with the base.

This is NOT ok.

$$x^b y^c \neq (x \cdot y)^{bc}$$

BRAIN POWER

Test it out with real numbers - try 3^2 and 4^3. Can you show that $(3^2)(4^3)$ is not the same as $((3)(4))^{(2)(3)}$ ***without*** working things all the way out to the answer?

there are no Dumb Questions

Q: Do I really have to memorize all of these rules for working with exponents?

A: No, because you can always work through these equations by working out each term separately. But if you can remember these rules, you'll be able to combine like terms and solve equations more quickly. It's much easier to combine terms and do one calculation. That's a lot better than working out a ton of terms separately, especially if the terms can be combined because they're like terms.

Q: What if the bases are different and the exponents are the same?

A: Well, there's a *little* bit you can do there. If the exponents are the same, then each term is being multiplied the same number of times, so you CAN mush them together, like this: $x^a \bullet y^a = (xy)^a$

It only works because of the commutative and associative properties. This is all just multiplication, so you can mix up the order, and it will still work.

Q: So what about that Brain Power? How could you show it without solving the math?

A: You can do it with variables, so instead of 3 and 4, let's use x and y. Then we have $x^2y^3 = x \bullet x \bullet y \bullet y \bullet y$ and

$(xy)^{(2)(3)} = (xy)^6 = xy \bullet xy \bullet xy \bullet xy \bullet xy \bullet xy = x \bullet x \bullet x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet y \bullet y \bullet y$

You can just look at those two and see they're not equal.

But what about all those hits I need? I need 734,166 more hits, and I only have 9 days left. I can't lose that sponsorship, or who knows how many subscribers I'll lose, too.

Addie needs another round of emails.

But how many does she need to send out? Addie has only 9 days left and she needs to figure out how many emails she needs to start with today to make up for the campaign that Movie Podcast's running.

We've got to work our exponent "backward"

Let's go back to Addie's equation. We've got different information this time: the number of hits we need and the number of days left:

Number of emails per day per person

no. of hits

The number of days

This is the general form of the original equation.

$$h = x^a$$

Now we fill in the things we know:

734,166

$$\cancel{h} = x^a$$

9 days left

$$734{,}166 = x^9$$

Now we need to solve for x.

What do I do with that? I can't solve for a base.

A root is the INVERSE of an exponent

We need an operation that can unravel an exponent. So when we have the exponent, what's the base to get a certain answer? Well, that's the **root**. When you find a root of something, you're finding the **number that can be multiplied** over and over to reach the final number.

For Addie, we need take the ninth root of both sides of her equation. That will isolate $\boldsymbol{x}$ and give us a numeric value on the other side.

Addie's equation.

$$\underbrace{734{,}166 = x^9}$$

We need the 9th root here.

This is the equation we started with

$$734,166 = x^9$$

$$\sqrt[9]{734,166} = \sqrt[9]{x^9}$$

$$\sqrt[9]{734,166} = x$$

We take the ninth root of both sides. This undoes the exponent of nine on the right.

We know this side needs to be x because the root and the exponent are the same, and they cancel each other out.

How did you figure out that crazy root? Do you have to be some sort of times-table genius?

Roots Up Close

It's time to pull out a calculator. Look closely—you can punch any root of any number in there and get a solution. Most calculators have a way to punch in both a root key and the root you want (like 9 or 3).

Ask a teacher, or look this up in your calculator's instructions.

Let's look at a little closer at roots:

The little (index) number shows how many times the root needs to be multiplied (in this case, 2).

This symbol is called the radical. It means find the root

$$\sqrt[2]{9} = 3$$

$3^2 = 9$ (See the connection?)

This is the root.

If you wanted to read this, you'd say, "The second root of nine is three." The actual root is three, and it's the number that you multiply twice by itself to find the number under the radical. So, to find the ninth root for Addie, all you need is a calculator.

Sharpen your pencil

It's the moment of truth. How many more emails does Addie need to send out? Is there any way she can pull this off?

Solve Addie's root: ....

....

....

How many emails does Addie need to send out? ....

....

Is there any way she can pull it off? **Yes** **No**

Why? ....

Sharpen your pencil Solution

It's the moment of truth. How many more emails does Addie need to send out? Is there any way she can do it?

Punch this into your calculator and you'll get a number you can use.

Solve Addie's root: $\sqrt[9]{734{,}166} = x$

$4.4849 = x$

This number goes on and on, but this is enough to get the idea.

How many emails does Addie need to send out? She needs to send out 5 emails in the first round.

This is another one of those situations where you need to think about the context of the problem. The answer here isn't 4.4849 emails.

She needs more than 4.4, so that's 5.

Is there any way she can pull it off? **Yes** **No**

Addie can also recruit more of Alex's friends if she needs people to send mail to.

Why? Sure – she's got to know 5 more people!

9 days later...

You've helped Addie land a big check!

Addie's site cleared 5,000,000 hits, no problem. Her sponsorship deals on, the subscribers are pouring in, and Addie's off to get some great new gear from her local Apple store. Next up... a video campaign on YouTube!

there are no Dumb Questions

Q: Just put the problem in a calculator? Is that for real?

A: There are actually several ways to find roots of numbers. There are tables where you can look them up, there's even a way to find them by hand that looks like long division. But honestly, they're all really old school. For most folks, a calculator is perfect.

Another way to get near the root of a number is to remember the perfect squares (2x2=4, 3x3=9,etc.). Then you can get an idea of what numbres might at least be close to what you're looking for.

Q: What's the inverse operation of exponents? The radical?

A: Not quite. It's finding the root. The radical is the symbol for the operation. It's just like the dot symbolizing multiplication.

Q: What if I see a radical without an index number?

A: Assume an index of 2. That's the square root. It's convention that if there isn't an index, then the equation is talking about the square root.

Q: Can you have a fractional exponent?

A: Yes. That simply means you should take the root of the base. For example, if you see 1/2 as an exponent, it means square root. 1/3 would be the third root, and so on.

Q: My calculator doesn't have a 9th root button, what do I do?

A: You can write a root as fractional exponent. So, a ninth root can be written as $\sqrt[9]{734{,}166}$ or $734{,}166^{(1/9)}$

Most calculators have an exponent button. So you could just put in a root of (1/9) and get the same answer.

Q: Will I ever need to solve for an exponent and not the base?

A: Not anytime soon. There are more operations out there that you can use to do this sort of problem, but they're well beyond this book. Don't worry about it for now. (Isn't that good to hear!)

Q: What about an exponent of 0?

A: Any number raised to the 0 power is one. Why? If you go back to the division of exponents, you subtract the bottom exponent from the top exponent. If you end up with the same term on the top and the bottom, then it's the base to the 0 power. That's always the number "1."

Q: What about an exponent of 1?

A: Any number raised to an exponent of 1 is itself. That means an exponent of one is implied over EVERY number and EVERY variable. It can come in handy sometimes to know that.

Q: Can an exponent be negative?

A: Yes - it means that it's the exponent in the denominator. So $X^{-2} = \frac{1}{X^2}$

That ties right in with subtracting exponents again. Since there's no exponent in the numerator, it's a negative exponent.

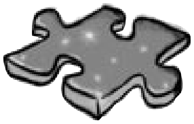
Q: Can you use negative exponents to get rid of fractions?

A: Yes. If you have an expression with fractions in it, you just rewrite the expression with the denominators as negative exponents. This really only helps if you find working with exponents easier than working with fractions. Of course, some people prefer that, and it's a perfectly okay way to work.

This also works the other way: if you think fractions are easier than exponents, just pull out all your negative exponents and rewrite them as fractions.

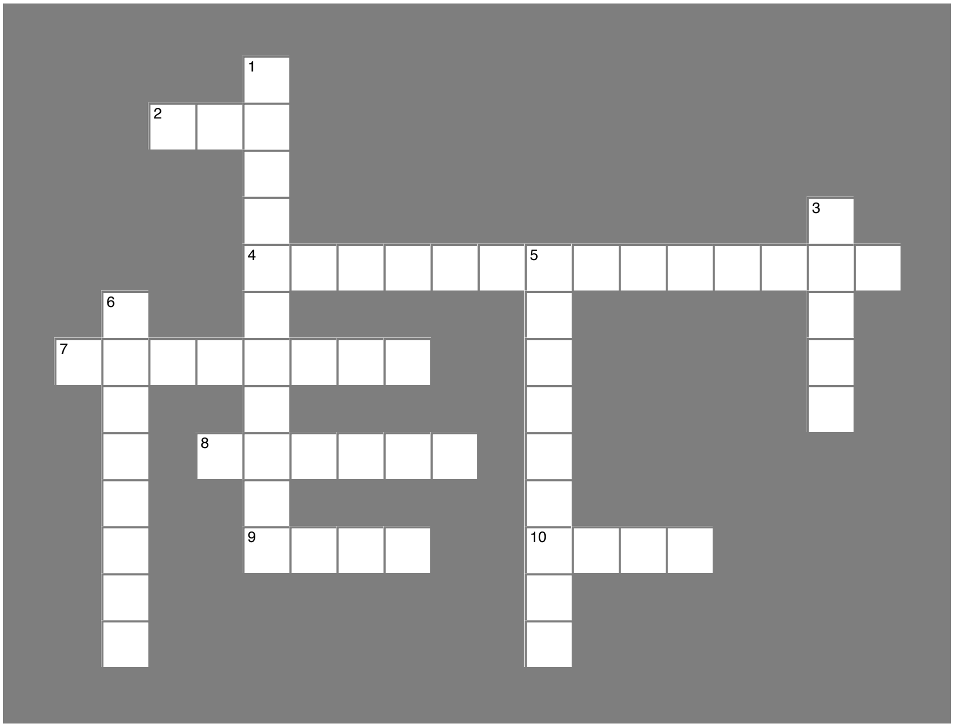
Q: I've heard of something called the principle root. What's that about?

A: When we talk about finding roots, we're actually talking about finding the **principle root**. That's the positive root of a value. There are other roots to numbers, too, though. The most common is the **negative root**. For example, the principle square root of 9 is 3, but -3 is a square root of 9, too, since (-3)(-3) = 9.



Exponentcross

Raise yourself! Can you get all the words?
They're all from this chapter.



Across

2. Any number raised to an exponent of zero is
4. Exponents are a faster form of
7. The number of times that the base gets multiplied is the
8. Any number raised to an exponent of one is
9. A fractional exponent is actually a
10. The inverse operation of an exponent is a

Down

1. A negative exponent means that the exponential term can be written as a
3. Another word used for exponent is
5. Exponents with the same base and exponent are
6. The number in an exponential term that gets multiplied

Who Does What?

We've written the exponent operations that we've been talking about in terms of general variables. Match each expression to it's simplification.

Variable problem	The simplified version
$x^a \cdot x^b$	$x^{a \cdot b}$
$(x^a)^b$	$2x^a$
$x^a - x^b$	x^{a+b}
$\frac{x^a}{x^b}$	x^a
$x^a + x^b$	x^{a-b}
$2x^a - x^a$	The expression is already simplified.
$\frac{x^a}{y^b}$	

WHO DOES WHAT? SOLUTION

We've written the exponent operations that we've been talking about in terms of general variables. Match each expression to it's simplification.

Variable problem

$x^a \cdot x^b$

$(x^a)^b$

Since these are not the same base and the same exponent, there's nothing else you can do

$x^a - x^b$

$\dfrac{x^a}{x^b}$

$x^a + x^b$

Just combine like terms.

$2x^a - x^a$

$\dfrac{x^a}{y^b}$

These are not the same base or the same exponent, so there's nothing you can do here, either.

The simplified version

Raising an exponent to an exponent is just more multiplication.

$x^{a \cdot b}$

$2x^a$

x^{a+b}

Here you add, remember?

x^a

x^{a-b}

Subtraction works here because you have the same base and the same exponent.

The expression is already simplified.

BE the calculator

Your job is to play calculator and crunch the numbers like your calculator would. You'll need to apply what you just learned about negative exponents and raising bases to zero. And since you're playing calculator, don't use one!

Just remember your exponent properties...

$1,467^0 + 1,856^1 =$ ....

....

....

Feel free to put in equals signs a few times – these could take a few steps.

$2^2 + 2^3 =$ ....

....

$2^2 \cdot 2^3 =$ ....

Try and come up with two ways to approach this one.

$\frac{5^7}{5^9} =$ ....

$=$ ....

There are a few different ways to do this one too – come up with two if you can.

$\frac{1}{3^3} + \frac{1}{3^3} =$ ....

$=$ ....

$=$ ....

BE the calculator

Your job is to play calculator and crunch the numbers like your calculator would. You'll need to apply what you just learned about negative exponents and raising bases to zero. And since you're playing calculator, don't use one!

Any number to zero is one.

Any number raised to one is itself.

$1,467^{0} + 1,856^{1} =$?

$1 + 1,856 =$ $1,857$

These numbers can't be added without being the same exponent and the same base.

All there is to do is simplify the exponents and then add them.

$2^{2} + 2^{3} =$ $2 \cdot 2 + 2 \cdot 2 \cdot 2 = 4 + 8 = 12$

These terms are the same base and different exponents, but that's ok since we're multiplying.

When you multiply exponents, you can add them up first.

$2^{2} \cdot 2^{3} =$ $2^{2+3} = 2^{5} = 32$

You didn't have to do all this, but there were a few right options.

The thing to remember to do here is subtract the exponents.

$\dfrac{5^{7}}{5^{9}} =$ $5^{7-9} = \dfrac{1}{5^{2}} = \dfrac{1}{25}$

Then, just treat that negative exponent as a denominator (fractional notation or decimal both work).

$=$ $5^{7-9} = 5^{-2} = 0.04$

The result is that you will get a negative exponent

There are a few different ways to do this one too.

Rewrite the fractions as negative exponents, and since they have the same base and the same exponent, they are like terms.

$\dfrac{1}{3^{3}} + \dfrac{1}{3^{3}} =$ $3^{-3} + 3^{-3} = 2(3^{-3}) = 2\left(\dfrac{1}{3^{3}}\right) = \dfrac{2}{27}$

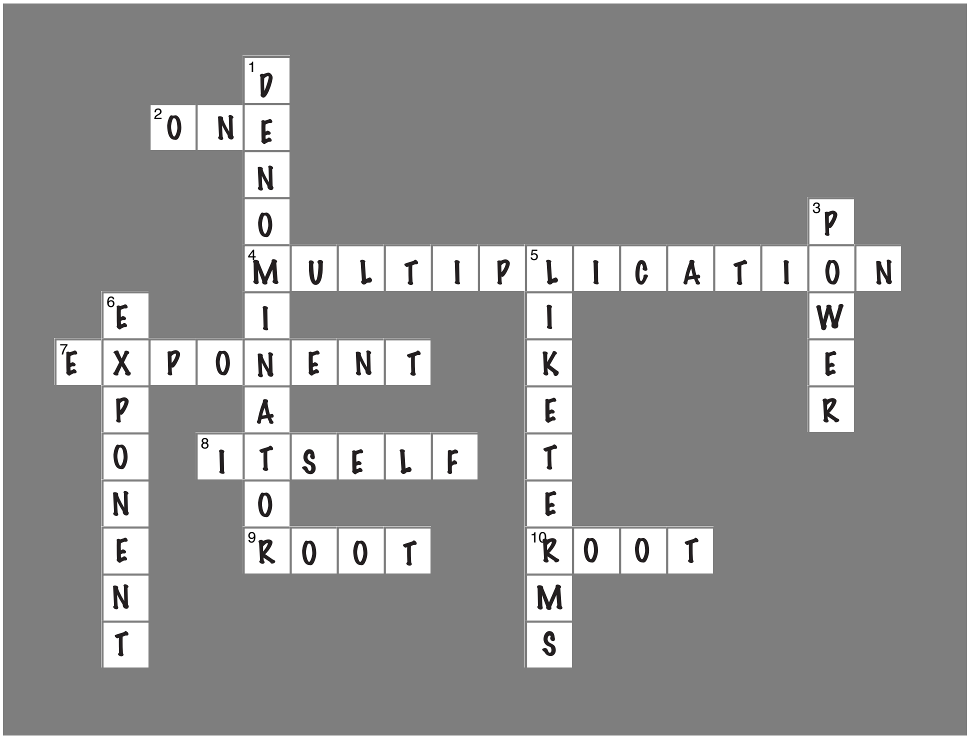
$=$ $\dfrac{2}{3^{3}} = \dfrac{2}{27}$

You can recognize them as like terms as fractions and add them first, then simplify.

$=$ $\dfrac{1}{27} + \dfrac{1}{27} = \dfrac{2}{27}$

You could simplify them first, without taking advantage of exponent rules.

Exponentcross Solution



Tools for your Algebra Toolbox

This chapter was about numeric properties that are important to understand in order to work with equations.

Exponential terms

exponent

$$x^{a} = x \cdot x \cdot x \ldots \cdot x$$

base

This means multiply x by itself "a" times.

$$x^a x^b = x^{a+b}$$

$$x^a y^a = (xy)^a$$

$$(x^a)^b = x^{ab}$$

$$\frac{x^a}{x^b} = x^{a-b} \text{ or}$$

$$\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$$

$$x^0 = 1$$

$$x^1 = x$$

$$x^{-a} = \frac{1}{x^a}$$

These are the general forms for exponential operations for exponential terms of the same base and different bases.

BULLET POINTS

- Exponents are shorthand for **repetitive multiplication**.
- The **base** is the number that gets multiplied.
- The **exponent** is how many times the base is multiplied.
- To **add** or **subtract** terms with exponents, they must have the **same base** and the **same exponent.**
- Adding and subtracting those terms is just combining **like terms**.
- To **multiply** exponential terms with the same base, just **add the exponents**.
- To **divide** exponential terms with the same base, **subtract the exponents**.
- To **raise** an exponential term to an exponent, **multiply the exponents**.
- Rules for dealing with exponents apply to **numbers** and **variables**.